Multiple-Choice Section

These answer explanations may not be entirely accurate, but they should be a good starting point. When in doubt, rely on your own judgment and problem-solving skills to ascertain the correct answer.

1. **(B)** // The unlabeled region with five squares (that looks like a staircase) <u>must</u> contain the number 3, because of the rule that adjacent regions cannot contain the same number. This invalidates **(C)**. Similarly, the unlabeled region with three squares <u>must</u> contain either a 1 or a 3. If the unlabeled region with three squares contains a 3, it is possible for Region A to contain either a 1 or a 2, which makes **(B)** correct.

2. **(C)** // It is possible for the knight to reach Square B on its third move, and reach Square A on its seventh move. For more information, search “how do chess knights move”.

3. **(A)** // All positive integers are either even or odd. If n is odd, then n^2 is also odd. But $n^2 + n$ (if n is odd) is equivalent to an odd number <u>plus</u> an odd number, which is an <u>even</u> number. By the same logic, if n is even, then n^2 is also even. Therefore, $n^2 + n$ (if n is even) is equivalent to an even number <u>plus</u> an even number, which is an even number. So for all positive integers n, $n^2 + n$ is an even number, which makes **(A)** correct.

4. **(D)** // Clue II informs us that the block of bronze can only be located in either Box A or Box C. To begin, assume that the block of bronze is in Box C. Since the blocks of silver and gold must be diagonally opposite each other (Clue I), the blocks of silver and gold must therefore occupy Boxes A and D. This, however, would mean that the BLT sandwich is located in Box B, which is a <u>contradiction with Clue III</u>. So the block of bronze must be located in Box A, and it can be determined that the BLT sandwich must be in Box D.

5. **(D)** // We know from Question 4 that the blocks of silver and gold must occupy Boxes B and C for every clue to be satisfied. But there is <u>no indication or clue</u> that allows us to determine if Box B contains the block of silver *and* Box C contains the block of gold or vice versa.

6. **(B)** // The code is encrypted with ROT13, a basic Caesar cipher which replaces a letter with the 13th letter after it (in the English alphabet). Once the code is cracked, the message reads: “trees are not waffles”, which makes (B) correct.

7. **(B)** // This is a simple combinatorics problem. There are 160 (8 x 2 x 10) total distinct orders, 16 of which (8 x 2 x 1) will spontaneously combust. 160 - 16 = 144, making **(B)** correct.

8. **(A)** // The knight can trap itself in 3 moves by moving to the bottom left corner, then making its one and only legal move (since its starting square is now lava), then moving to the top right corner (and trapping itself).

9. **(B)** // The knight will successfully trap itself if it can move to the square exactly two squares above its starting point. This can be accomplished in just two moves.

10. **(D)** // Solve the puzzle using the normal rules of Sudoku. There is only one solution, and in that solution, 'A' must be the number 4.

11. **(B)** // Consider the nth letter in the sequence. If n is odd, then the nth number is n^2. If n is even, the nth number is 10n. Therefore, the 7th number is 7^2, or 49, and the 8th number is 10(8), or 80. 49 + 80 = 129.

12. **(A)** // This is a classic variation of a riddle, except the original riddle concerns 1000 bottles of wine and 10 prisoners. Generally, you can test up to 2^x bottles with *x* mice. For a detailed explanation, search '1000 bottles of wine riddle' on Google.

13. **(A)** // Ordinarily, a chess bishop can only visit half of the squares on a chessboard, because they can only move diagonally. However, this special bishop can move to all of the squares on the board, because by simply moving down one square, it can access a completely new set of squares.

14. **(B)** // There are two distinct paths for the bishop that take 5 moves. One includes visiting the closest O first, then the next closest O, then the final O. There is no path that takes 4 or less moves.

15. **(B)** // There are 16 total squares. Before the king starts moving, there are 3 lava pit squares and 1 square that already has a piece of chocolate on it. The king can successfully visit the remaining 12 squares without 'wasting' a move (ie. by placing more than one piece of chocolate on each square). Therefore, the answer is **(B)**.

16. **(C)** // When the knight finally becomes trapped, there is only one square that is not lava. This is because squares that the knight leaves turns into lava.

Scoring Guidelines for the Free-Response Section on the next page →

Free-Response Scoring Guidelines

These answer explanations may not be entirely accurate, but they should be a good starting point. When in doubt, rely on your own judgment and problem-solving skills to ascertain the correct answer.

Question 1 (6 points total; suggested time: 8 minutes)

(a.i) One (1) point is awarded for correctly identifying that zero (0) logicians will guess their own hat color correctly. Each logician is wearing a blue hat, and can see that the other two logicians are wearing blue hats. After hearing Sasha's statement, each logician will <u>incorrectly</u> guess that their hat is red.

(a.ii) One (1) point is awarded for indicating (roughly) that "I, as a logician, will see two blue hats." And one (1) point is awarded for indicating (roughly) that "Since I see two other blue hats, I will conclude that my hat must be red." Even if the answer does not include these statements, full points <u>may be awarded</u> as long as the entire thought process (and guess) of the logician is made clear.

(b) One (1) point is awarded for explaining that <u>only</u> the logician wearing the blue hat will be skeptical of Sasha's statement. And one (1) point is awarded for a correct explanation of <u>why</u> only the logician wearing the blue hat will be skeptical.

(c) One (1) point is awarded for indicating '0.5x'. This answer is best explained by assuming that x = 20 (although this holds true for any positive even number). If x = 20, then there will be 10 logicians wearing blue hats and 10 logicians wearing red hats. Sasha will tell all of the logicians that there are 9 blue hats and 11 red hats. All of the logicians who wear red hats will be skeptical of Sasha's statement, since they know that there are at least 10 blue hats. None of the logicians wearing blue hats will be skeptical of Sasha's statement. Therefore, half of the logicians (0.5x) will be skeptical of Sasha's statement.

(a.i) – 1 possible point
(a.ii) – 2 possible points
(b) – 2 possible points
(c) – 1 possible point

Total: 6 possible points

On the next page: Scoring Guidelines for Question 2 →

Question 2 (8 points total; suggested time: 12 minutes)

(a) Two (2) points are awarded for indicating that there are 20 distinct paths. The easiest way to solve this entire problem: start by labeling A1, B1, C1, D2, D3, and D4 with the number '1'. Each square should contain the number of distinct ways that the rook can get to Square N if it starts on that square. (So in part a, C2 would have '2', C3 would have '3', and B3 would have '6'. Continue to fill in the grid with numbers, and once you find the number on Square A4, you're done!

(b) One (1) point is awarded for indicating that there are 12 distinct paths that include the pizza parlor.

(c.i) One (1) point is awarded for indicating the square 'b2'. And one (1) point is awarded for indicating the square 'c3'.

(c.ii) One (1) point is awarded for indicating the square 'b3'.

(d) Two (2) points are awarded for indicating that there are 35 distinct paths.

(a) – 2 possible points
(b) – 1 possible point
(c.i) – 2 possible points
(c.ii) – 1 possible point
(d) – 2 possible points

Total: 8 possible points