AP Puzzles
Section I — 40% of Exam Score
Time — 20 minutes
2 Questions

Question 1 (6 points total; suggested time: 8 minutes)

For all parts of this question: The logicians use perfect reasoning, cannot communicate, cannot see the color of their own hat, but can see the color of all other hats.

(a) Three perfect logicians are given either a blue hat or a red hat to wear. They cannot see the color of their own hat, but they can see the color of the other hats. However, the three logicians are separated by glass walls and cannot communicate in any way.

Sasha lies and tells each of the logicians: "There are, in total, 2 blue hats and 1 red hat." In actuality, there are (in total) 3 blue hats and 0 red hats – but the logicians are not aware of this fact.

After hearing Sasha's statement, each of the logicians individually tells Sasha what color they think their own hat is, based on their observations.

(a.i) How many logicians will guess their own hat color correctly?

(a.ii) You are one of the logicians. Explain your thought process when guessing the color of your own hat. You are not aware that Sasha lied.

(b) Sasha disposes of each logician's hat and places a new hat on each logician's head. Of the new hats, 1 is blue and 2 are red.

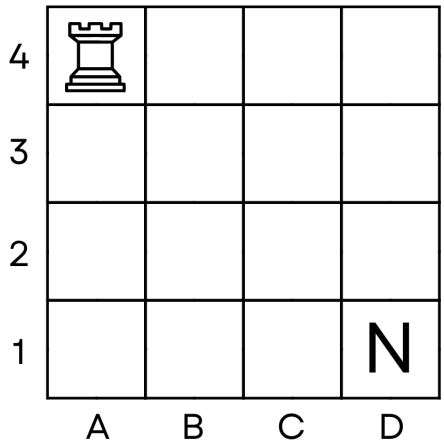
Sasha lies again and tells each of the logicians: "There are, in total, 2 blue hats and 1 red hat." Which logicians will be skeptical of Sasha's statement, and why? Explain your answer.

(c) There are now x logicians, where x is a positive even number. Half of the logicians wear red hats, and the other half wear blue hats. Sasha lies and tells each of the logicians: "There are, in total, $0.5x - 1$ blue hats and $0.5x + 1$ red hats."

How many logicians will be skeptical of Sasha's statement in part (c)? Express your answer in terms of x.

Question 2 (8 points total; suggested time: 12 minutes)

A rook is located at the top left corner of a 4×4 chessboard, as shown. The rook starts at Square A4, and Square D1 contains an ‘N’.



At any given point, the rook can only do one of two moves:

1. Move exactly 1 square down.
2. Move exactly 1 square right.

(a) How many distinct paths can the rook take from A4 to Square N?

(b) There is a pizza parlor located at C2, and the rook really wants pizza on its journey to Square N. How many distinct paths can the rook take from A4 to D1 if the rook’s path must include C2?

(c) You are allowed to turn any two squares on the board into lava (even square C2). The rook’s path cannot include any squares that are lava.

(c.i) Which two squares, when turned to lava, will only allow the rook to take two distinct paths from A4 to D1? List the two squares using the coordinate system.

(c.ii) The two squares you turned into lava in part (c.i) turn back into normal squares. Now, squares B1 and B4 are turned into lava. Identify another square, that, if turned into lava, would only allow the rook to take two distinct paths from A4 to D1.

(d) A full column of squares is added to the right of the 4×4 chessboard. This means that the squares E1, E2, E3, and E4 now exist. The letter N is moved to square E1. Now, how many distinct paths can the rook take from A4 to Square N?